

Correlation functions for a WZW theory and representation of quasitriangular Hopf algebras — Part one

Vassilis Konstantopoulos
August 2016

Abstract. For a finite dimensional complex Simple Lie algebra g we consider the associated standard Hopf QUE algebra, we examine the equivalence according to the theory of Kohno and Drinfeld of the two representations of the braid group ρ^{KZ} and ρ^{QYBE} . We consider the KZ equation as derived in the WZW conformal field theory and in the affine Lie algebras theory.

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Conclusion

1. There exists a connection between quantum groups and affine Lie algebras, the KZ equation is intimately related to representations of affine Lie algebras and in the Kohno-Drinfeld monodromy theorem we see that the KZ equation is also related to QUE algebras.

2. The KZ equation first appeared in Conformal field theory as the equation satisfied by the “ m -point” functions of the theory, by considering the monodromy of its solutions along closed paths, it leads to representations of braid groups.

3. If $U_h(g)$ is the standard QUE algebra associated to a complex simple Lie algebra g , and R_h is its universal R -matrix, the braid group representation defined by R_h is equivalent to that defined by the KZ equation.

The two braid group representations arise essentially from a solution of QYBE and a solution of the CYBE, respectively, and indeed the proof proceeds by using the KZ equation to construct a quantization of a certain quasi-Lie bialgebra.

4. The correlation functions for a rational WZW theory form a representation module for a quasitriangular Hopf algebra constructed by the Lie algebra defined in the WZW model.