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Symbol Index

ACKERMANN(x) Ackermann function 81
AP arithmetic progression 93, 752
B(l,r) upper bound of van der Waerden number W(l,r) 52
b(m,n) upper bound of Ramsey number r(m,n) 15
C_t^n the n-cube over t elements 44
Conj 7(m,n) 149, 150
Conj 11(k,r) 160
$c(2P_j^i)_a$ 896
$c(2P_j^i)$ 898
$c(2P_j^1, 2P_j^2, \cdots, 2P_j^i)$ 898, 1128
$c(+2P_j^i)$ 898, 899, 1129
$c(2P_1^i, 2P_2^i, \cdots, 2P_j^i)$ 1129
$c[+2P_j^i]$ 1129
degx degree of vertex x 8
DOUBLE(x) double function 79, 80
E(G) edge set of a graph G 7
EXPONENT(x) exponent function 80
f.p. fractional part 391
$f_\omega(x)$ Ackermann function 81
$\bar{G}$ the complement of graph G 8
gcd greatest common divisor 278
HJ(r,t) Hales-Jewett number 45
K_p complete graph of order p 10
L length of number 1069
l-AP l-term arithmetic progression 93
[n] 39
$[n]^k$ 39

$n \to (l_1, l_2, \cdots, l_r)$ arrow notation 40

$n \to (l)_r$ arrow notation 40

$n \to (l_1, l_2, \cdots, l_r)^K$ arrow notation 41

$n \to (l)^K_r$ arrow notation 41

$N(x)$ neighbourhood of vertex x 8

$nz[+2P_j^i]$ 1131

$nz(+2P_j^i)$ 1131

(p,q) graph a graph of order p and size q 8

(PTK) Proof Table K 1257

$R(l_1, l_2, \cdots, l_r)$ Ramsey function 40

$R(l; r)$ Ramsey function 40

$R(l)$ Ramsey function 40

$R_K(l_1, l_2, \cdots, l_r)$ Ramsey function 41

$R_K(l; r)$ Ramsey function 42

$R_K(l)$ Ramsey function 42

$r(m,n)$ Ramsey number 12

$R(m,n)$ Ramsey number 42

$r(n_1, n_2, \cdots, n_K)$ Ramsey number 38

$str(3,n)$ structure (3,n) 124

$\Sigma str(3,n)$ sum of digits of $str(3,n)$ 125

$str(m,n)$ structure (m,n) 138-140

$\Sigma str(m,n)$ sum of digits of $str(m,n)$ 141, 145

$TOWER(x)$ tower function 80, 85

$V(G)$ vertex set of a graph G 7

$v(a)$ numerical value of string a 1229

$W(l, r)$ van der Waerden number 44, 100

$W(l)$ van der Waerden number 44, 101

$W(l_1, l_2, \cdots, l_r; r)$ van der Waerden number 93, 100

$W'(l,r)$ upper bound of van der Waerden number $W(l,r)$ 51

WOW wow function 81

$|X|$ cardinality of set X 39

$[X]^{\kappa}$ 39

Z_{p_i} 642

$z[+2P_j^i]$ 1130

$z(+2P_j^i)$ 1131

$\sum_m n$ sum of order m 21,226,230,231

$\binom{\mu}{\nu}$ Binomial coefficient 24,230,241,469

$a|b$ a divides b 356

$\overset{c}{+}$ cyclic sum 398

$\overset{c}{-}$ cyclic difference 398

$=.$ equality of fractional (or of decimal) parts 437

$.a$ fractional part of a 489

$a \nmid b$ a does not divide b 511

$x_1 \equiv x_2 \pmod m$ x_1 is congruent to x_2 modulo m 587

NP_a^m progression of length N, with step a, starting at point m from the right end of the sequence 767

2^n set $\{1,2,\cdots,2^n\}$ 785

$2_n^{\cdot}$ 785

$A+a,\cdot n$ 796

$(A_{[B]}C)$ 799, 801–808

$(2^{a-1}{}_{[2^m]}2^{m-1})$ 799, 801–808

$a-\{c_1,c_2,\cdots,c_\lambda\}$ 816

$\{\bullet NP_a^m\}$ 851

$\{\circ NP_a^m\}$ 851

$\{\overset{\bullet}{\circ} NP_a^m\}$ 851

$\{NP_a^m\}$ 851

A_a restriction of set A by integer a 852, 853

$\{\bullet NP_a^m\}_a$ restriction of set $\{\bullet NP_a^m\}$ by integer a 853

$\{\circ NP_a^m\}_a$ restriction of set $\{\circ NP_a^m\}$ by integer a 853

$\{NP_a^m\}_a$ restriction of set $\{NP_a^m\}$ by integer a 853

$U(A_1, A_2, \cdots, A_K)$ union of sets $A_1, A_2, \cdots, A_K$ 853

$\overset{\bullet}{\circ} 2P_a^m$ Blue and red progressions $2P_a^m$ 867

$\{NP_a^m\}'$ subset of set $\{NP_a^m\}$ 885

$\{NP_a^m\}_a'$ subset of set $\{NP_a^m\}_a$ 885

$2P_j^i$ $2P_a^m$ with i, j instead of m, a 895

$(2P_j^i)_a$ 896

$(2P_j^i)$ 898

$(2P_j^1, 2P_j^2, \cdots, 2P_j^i)$ 898, 1128

$(+2P_j^i)$ 898, 899, 1128, 1131

$\mathbb{O}$ 2^K-point digit $00\cdots0$ 926, 917, 920–921, 922

$\mathbb{1}$ 2^K-point digit $11\cdots1$ 926, 917, 920–921, 922

$(2P_1^i, 2P_2^i, \cdots, 2P_j^i)$ 1129

$[+2P_j^i]$ 1129, 1131

$\equiv$ (in Ch. 7) By definition 1172, 1177

$\lfloor x \rfloor$	integer part of number x 1239
$*$	asterisk 1259, 1266-1268
$\bar{*}$	overlined asterisk 1259, 1266-1268
$\overset{*}{m}$	1259, 1267
$k\overset{*}{l} \cdot \cdot \cdot m$	1260, 1267
$a_{\underline{1}}^{2\rho+1}$	1263
$a_{\underline{k+1}}$	1264 - 1265
$\{a \underset{\{\longleftarrow\}}{\text{string}}\}$	left-arrow below a string 1268-1269, 1306

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